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WALL INFLUENCE ON THE AERODYNAMIC CHARACTERISTICS OF AN OSCILLATING AIRFOIL

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UDC 533.6

The difference between the aerodynamic characteristics of an airfoil in an unbounded fluid and an airfoil in the neighborhood of a wall is of great practical interest. It is of interest not only in the design of transport vehicles using wings as lifting surfaces but also in the development of new propulsive systems using flapping wings [1]. Computations on the unsteady aerodynamic characteristics of airfoils in the neighborhood of a solid boundary have been carried out in a number of papers, e.g., [2-4]. A fairly comprehensive review of literature in this field is available in [5, 6]. The common feature in all these methods [2-6] is that they have been carried out within the framework of linear theory for thin airfoils with small camber. There are very few independent results for the nonlinear problem (see, e.g., [5, 6]) but even they are only for the case of an airfoil moving extremely close to the wall or under steady-state conditions. The nonlinear problem of the flapping motion of a thin airfoil in the neighborhood of a solid plane wall in an ideal incompressible fluid is investigated in this paper. In this nonlinear problem the shape of the vortex sheet behind the airfoil is not specified initially but is determined in the course of the solution. The problem has been solved by the method of discrete vortices [7].

1. Consider the motion of a thin airfoil in an ideal, incompressible fluid on a solid, plane boundary. We introduce a Cartesian coordinate system $O_1x_1y_1$ (nondimensionalized with respect to the chord length) in which the fluid is at rest at infinity. Let at time $\tau = 0$ the airfoil start from rest with a specified initial velocity $\vec{V}(x_1, y_1, t)$, where $t = V_0\tau/b$, and V_0 is a certain characteristic speed (e.g., $V_0 = |V(\tau_*)|$, $\tau_* > 0$). The airfoil is replaced by an infinitely thin plate $S_0(t)$, assuming the effect of thickness to be negligible. The vortex wake behind the plate is denoted by $S_1(t)$. The fluid motion outside the contour $S = S_0 \cup S_1$ is assumed to be potential.

The contour $S(t)$ is modeled by a vortex sheet of strength $\gamma = v_{\sigma-} - v_{\sigma+}$, and the pressure jump across the point $M \in S(t)$ will be determined by the Cauchy–Lagrange integral

$$\frac{p_- - p_+}{\rho V_0^2} = -\frac{\partial}{\partial t}\int_0^s \gamma(\sigma, t)\, d\sigma - \gamma(s, t)(v_{0\sigma} - v_{e\sigma}), \qquad (1.1)$$

where the positive and negative signs represent the limiting values of the functions when approaching the contour $S(t)$ from above and below, respectively; the index denotes the projection of the vector onto the unit tangent to $S(t)$ in the direction of increasing s; s is the arc abcissa of the point $M \in S(t)$, measured from the leading edge of the plate; ρ is the fluid density; $\vec{v}_0 = (\vec{v}_+ + \vec{v}_-)/2$; $\vec{v}_e$ is the translational velocity of the point M.

Along with the stationary coordinate system $O_1x_1y_1$, a body-fitted moving system of Cartesian coordinates Oxy is introduced to solve the problem. The x axis is along the chord

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from the leading edge. Let us assume that at every instant t the contour $S(t)$ is smooth in the Lyapunov sense and the function $\gamma(s, t)$ on it belongs to the H^* [8] class in the neighborhood of the leading edge of the contour. This allows us to determine the velocity at any point in the fluid and at the point $M \in S(t)$ using the well-known Biot–Savart law. The resulting velocity field is potential outside $S(t)$ and the disturbance velocities damp out at infinity everywhere outside $S_1(t)$. Satisfying the remaining boundary conditions of the problem of the motion of the thin airfoil close to the plane wall (see, e.g., [6]), the following system of equations is obtained for the vortex strengths γ_0, γ_1 on the contours $S_0(t)$, $S_1(t)$:

$$\int_0^1 \gamma_0(\xi, t)\left\{\frac{1}{x-\xi} + G_0(x, \xi, h)\right\} d\xi = 2\pi V_y(x, t) - \int_{S_1(t)} \gamma_1(\sigma, t)\left\{\frac{x-\xi(\sigma, t)}{(x-\xi)^2+\eta^2} + G_1(x, \xi, \eta, h)\right\} d\sigma \tag{1.2}$$

for $x \in (0, 1)$;

$$\frac{\partial \mathbf{r}}{\partial t} = \mathbf{v}_0(\mathbf{r}, t),\ \mathbf{r}(\gamma, t_\gamma) = \mathbf{r}_0(\gamma); \tag{1.3}$$

$$\int_{s_1(t)}^{s_2(t)} \gamma(\sigma, t)\, d\sigma = \Phi(s_1, s_2, t); \tag{1.4}$$

$$\frac{d}{dt}\int_0^1 \gamma_0(x, t)\, dx = -w_x(1, t)\gamma_1(1, t),\ \mathbf{w} = \mathbf{v}_0 - \mathbf{V} \tag{1.5}$$

for $M \in S_1(t)$. Here $\vec{r} = \vec{r}(\gamma, t)$ is the radius vector of the points in the vortex sheet wake $S_1(t)$, which is assumed to be a function of vorticity and time t; t_γ is the moment the vortex γ leaves the trailing edge of the airfoil; $\vec{r}_0(\gamma)$ is the radius vector of this vortex at $t = t_\gamma$; the quantity $\Phi(s_1, s_2, t)$ is determined at the moment the vortex element (s_1, s_2) is formed and remains constant for fixed values of s_1, s_2 on $S_1(t)$, though the element itself is deformed in accordance with the change in the velocity field; functions G_0 and G_1 take into consideration the influence of the wall and are obtained by taking the mirror image of the airfoil $S_0(t)$ and the vortex wake $S_1(t)$ with respect to the solid wall $y_1 = 0$, with the vortex strengths γ_l replaced by their opposites: $-\gamma_l$ $(l = 0.1)$.

Since the flow segment and the velocity vector $\vec{V}$ of the motion of the points on the airfoil depend on time, the system (1.2)-(1.5) has to be solved with the initial conditions, which in the case of motion from rest take the form

$$S(0) = S_0(0),\ \gamma(x, 0) = 0. \tag{1.6}$$

If the airfoil is subject to small, steady oscillations (linear problem) the initial conditions (1.6) are unnecessary and the system (1.2)-(1.5) is simplified. The consideration of nonlinear effects associated with the deformation of the vortex wake (Eqs. (1.3), (1.4)) behind the oscillating airfoil makes it possible to solve the system (1.2)-(1.5) only approximately. Its solution in this case is sought at a number of instants of time t_n, starting from $t_0 = 0$ when the condition (1.6) is satisfied, using the method of discrete vortices [7].

The airfoil is divided into N elements (x_{k-1}, x_k) each with a vortex strength

$$\Gamma_k^{(n+1)} = \int_{x_{k-1}}^{x_k} \gamma(x, t_{n+1})\, dx.$$

The solution of the discretized system of equations (1.2)-(1.5) determines the quantities $\Gamma_k^{(n+1)}$, starting from $n = 0$. These values are used to determine the continuous vortex layer γ_0 on S_0 [9], which is necessary both for computing the load distribution on the airfoil and also for the determination of the suction forces on it.

2. The normal force P_q (referred to $\rho V_0^2 b/2$) acting on the element $S_{0q} = \{x : x_{q-1} \leqslant x \leqslant x_q\}$, is expressed (in accordance with (1.1) $(\vec{v}_e = \vec{V})$) in the form

$$P_q = \int_{S_{0q}} dP = P_{q1} + P_{qt} + P_{qit},$$

where P_{q1} determines that part of the force which depends on γ_0 and the quantities P_{qt}, P_{qit}

are associated with the variation in circulation around the airfoil

$$P_{qt} = -\frac{2}{N}\frac{d}{dt}\sum_{k=1}^{q-1}\Gamma_k^{(n+1)},\; P_{qit} = -2\frac{d}{dt}\int\limits_{S_{0q}}\left(\int\limits_{x_{q-1}}^{x}\gamma_0 d\xi\right)dx.$$

Neglecting the quantities of the order of Γ_q/N and higher, we get,

$$P_{q1} = -W_{qx}\Gamma_q^{(n+1)},\; P_{qit} = -\frac{2}{N}(1-\mu_q)\frac{d}{dt}\Gamma_q^{(n+1)},$$

where the coefficient μ_q determines the location ($x_q = (q - 1 + \mu_q)/N$) of the discrete vortex Γ_q on the element S_{0q} as a fraction of its length, and

$$W_{qx} = 2w_x(x_{0q}, t_{n+1}),\; x_{0q} = (q - 0.5)/N. \tag{2.1}$$

The suction force Q (referred to $\rho V_0^2 b/2$) is obtained using the momentum theorem applied to the fluid inside a circle of radius $\varepsilon \ll 1$ with the center at the airfoil leading edge. It is possible to show that as $\varepsilon \to 0$, $Q = -\pi a^2/2$, where a is the coefficient of the strength of the vortex layer at the singularity $x^{-1/2}$. An approximation of the vortex layer, suggested in [9], makes it possible to compute a using Γ_q^{n+1}.

Nondimensional coefficients of the normal force P and the suction force Q are determined as follows:

$$c_n = P = \sum_{k=1}^{N} P_k,\quad c_q = Q.$$

3. For the practical realization of the method described in Sec. 2, the algorithm for the computation of the aerodynamic characteristics at the (n + 1)-th time step is conditionally divided into a number of stages: 1) the selection of the step size in time Δt_{n+1}; 2) the determination of the coordinates $\vec{r}_q^{(n+1)}$ of the vortex wake S_1 from the solution of the Cauchy problem (1.3) for the q-th free vortex

$$\mathbf{r}_q^{(n+1)} = \mathbf{r}_q^{(n)} + \mathbf{w}_{1q}^{(n)}\Delta t_{n+1},$$

where

$$\mathbf{w}_{1q}^{(n)} = \begin{cases}\mathbf{w}_q^{(n)}\left(1 + \dfrac{\Delta t_{n+1}}{2\Delta t_n}\right) - \mathbf{w}_q^{(n-1)}\dfrac{\Delta t_{n+1}}{2\Delta t_n},\; q \leqslant n,\\ \mathbf{w}_q^{(n)},\; q = n+1;\end{cases} \tag{3.1}$$

3) computation of the velocity field $\vec{w}_q^{(n+1)}$ at the given points (2.1) on the airfoil S_0 and the vortex wake $S_1(t_{n+1})$; 4) determination of the coefficients c_n, c_q.

Let us consider some of these stages. Following [7], the time step $\Delta t_{n+1} = t_{n+1} - t_n$ is selected from the condition

$$\Delta t_{n+1} = 1/(Nw_x(1, t_n)). \tag{3.2}$$

Condition (3.2) ensures uniform distribution of vortices in the neighborhood of the airfoil trailing edge. The coordinates of the points on the vortex wake were determined, unlike [7, 9], using the second-order difference scheme (3.1).

The convergence of the numerical scheme was verified numerically by comparing the computed results with different number N of vortices on the airfoil. Computational results with time step Δt_n and a time step half its value (in view of (3.2) this corresponds to the condition when the number of elements on the airfoil equals N in one case and then in the other case it is 2N), were practically the same. This leads to the conclusion on convergence in the given case.

In order to verify the algorithm for the computation of the aerodynamic characteristics of the airfoil in the neighborhood of the wall, its steady motion with velocity V = const = 1 along the wall at a constant angle of attack α is considered. The following results have been observed for such an airfoil motion: firstly, unsteady values of the aerodynamic characteristics monotonically approach certain values which were later chosen as the steady state values corresponding to the particular angle of attack; secondly, such a computation of these characteristics is quite economical since they are stabilized before the airfoil covers 3-4 chord lengths.

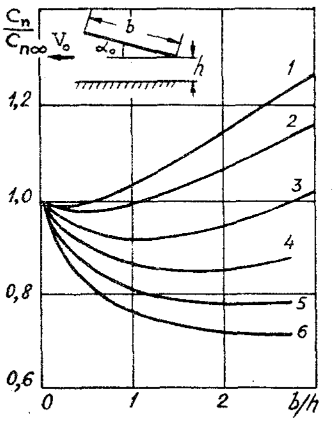

Fig. 1.

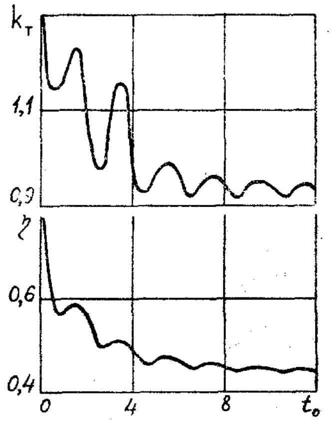

Fig. 2.

The relation between the ratio of the normal force coefficient c_n, obtained from the computation, to the quantity c_n which corresponds to the case of the airfoil motion in an unbounded fluid, and the parameter b/h is shown in Fig. 1. Here h is the distance from the airfoil trailing edge to the wall at different angles of attack (α = 2, 5, 10, 15, 20, and 25° for the curves 1-6, respectively). It is seen that the presence of the solid wall at large angles of attack leads to a reduction in c_n. At relatively short distances to the wall and at fairly low angles of attack, the coefficient c_n increases as the wall is approached. This result agrees with L. I. Sedov's conclusion [6].

The investigation of unsteady aerodynamic characteristics of the airfoil close to the wall was carried out for the longitudinal oscillations of the airfoil given by

$$y(t) = (y_0/b)\cos kt, \tag{3.3}$$

where y_0/b is the nondimensional amplitude of oscillations; $k = \omega b/V_0$ is the Strouhal number. In addition to the coefficients c_n, c_q we determine the power required to maintain the oscillations (3.3):

$$N_0(t) = -\rho V_0^3 b \int\limits_0^1 \Delta p \dot{y} dx,$$

where $\dot{y}$ is the nondimensional frequency of oscillations, and the thrust coefficient $c_T = -c_q$. For the practical application of the flapping airfoil as a means of propulsion, it is necessary to get the mean value of the coefficients over a period of oscillations $T = 2\pi/k$:

$$\bar{c}_T = \frac{1}{T}\int\limits_{t_0}^{t_0+T} c_T(t)\,dt,\ \overline{N}_0 = \frac{1}{T}\int\limits_{t_0}^{t_0+T} N_0(t)\,dt. \tag{3.4}$$

The efficiency is determined on the basis of the averaged quantities $\bar{c}_T$ and $\overline{N}_0$:

$$\eta = \rho V_0^3 b\bar{c}_T/(2\overline{N}_0). \tag{3.5}$$

In connection with the computation of the quantities (3.4), (3.5) using the above-described algorithm to solve the nonlinear problem, the following situation may be observed. It is known that the result of averaging any periodic function (with period T) does not depend on the point t_0. However, a different situation arises in the determination of averaged aerodynamic characteristics obtained from the solution of the nonlinear problem of an oscillating airfoil. The dependence of the efficiency η and the normalized thrust coefficient [1]

$$k_T = \frac{c_T}{k^2\left(\frac{y_0}{b}\right)^2},\ \left(k_T = \frac{R_x}{\frac{1}{2}\rho\,(\omega y_0)^2\, b}\right)$$

on the initial point of averaging when $k = \pi$, $b/h = 0.5$, and $y_0/b = 0.25$ are shown in Fig. 2.

The behavior of the curves is explained by the strong influence of transitional processes on these quantities at the start of the motion from rest. It is worth mentioning that in an unbounded fluid ($h = \infty$) this effect of transitional process is extended even more. Thus, for the same parameters $y_0/b = 0.25$ and $k = \pi$, but, $b/h = 0$, the dependence of k_T on t_0

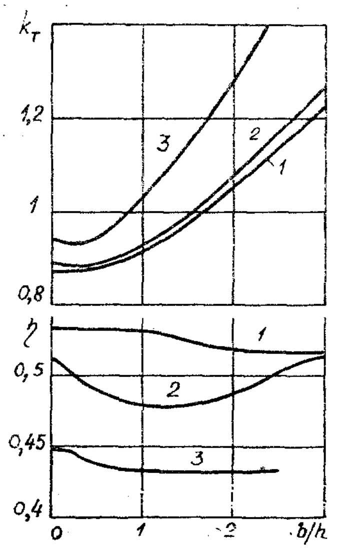

Fig. 3.

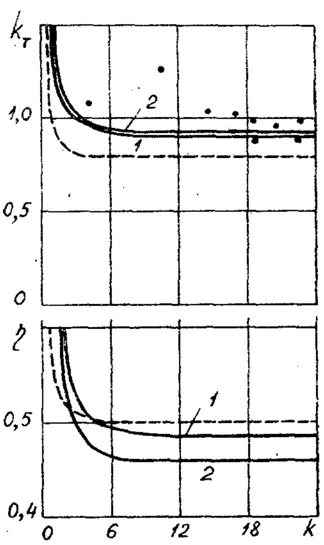

Fig. 4.

is maintained up to $t_0 = 18$, for η it is up to $t_0 \approx 15$, whereas when $b/h = 0.5$ values of t_0 are 14 and 12, respectively. Such a difference can be, apparently, attributed to the stabilizing action of the wall on the vortex wake behind the oscillating airfoil which in turn leads to a reduction of its influence on the aerodynamic characteristics.

In view of the above-described situation, the coefficient k_T and efficiency η obtained after a repeated averaging with equations of the type (3.4) with a period $T = 2\pi/k$ were then taken as mean values of k_T and η. Here the initial point t_0 for the repeated averaging was chosen such that the error δ in determining these quantities was less than 10^{-4} when t_0 is increased by 0.1T. Results of the computation shown in Fig. 2 indicate that the consideration of 7-8 periods of oscillations are sufficient in the given case to determine the quantities k_T and η.

4. Consider now some of the results of computations. The dependence of the coefficients k_T and efficiency on b/h for a Strouhal number $k = \pi$ and different amplitudes of longitudinal oscillations is shown in Fig. 3. It is seen that the presence of the wall leads to an increase in the thrust coefficient k_T for all h when $y_0/b = 0.01$ (curve 1) and $h > 0.5$, and for $y_0/b \geqslant 0.1$ (curves 2, 3). The efficiency is reduced because of the increase in power required to maintain the oscillations at shorter distances from the wall.

It may also be mentioned that the computation of the aerodynamic characteristics of the oscillating airfoil in an unbounded fluid ($b/h = 0$) showed a fairly strong influence of the amplitude of oscillations y_0/b on the thrust coefficient k_T, which apparently limits the validity of the linear theory for $y_0 > 0.1b$.

The results of the computation of the effect of the wall on the dependence of the thrust coefficient and efficiency on the Strouhal number are shown in Fig. 4. The solid lines indicate the computed values of the coefficients for $b/h = 1.67$ and amplitudes $y_0/b = 0.01$; 0.08 (curves 1, 2 respectively), the dashed lines refer to the values of the coefficients obtained from the linear theory [10] for an unbounded fluid ($b/h = 0$). The solid dots in the same figure indicate the experimental results for $y_0/b = 0.08$, $b/h = 1.67$ (courtesy D. N. Gorelov and A. V. Piner). It is seen that the experimental data agree fairly well with the theory.

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SPATIAL ANALOG OF CENTERED RIEMANN AND PRANDTL–MEYER WAVES

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In this paper, we prove the existence of solutions of equations of spatial gasdynamics that have special properties: waves, centered on arbitrary two-dimensional surfaces in four-dimensional space **x**, t. These solutions are generalizations of the centered Riemann waves in the theory of one-dimensional nonstationary motion and centered Prandtl–Meyer waves in the theory of planar stationary flows. Characteristics of this form arise in problems of the interaction of shock waves with fronts having arbitrary shapes, interaction of shock waves and a contact discontinuity, and piston problems.

1. Formulation of the Problem. We are examining equations that describe the spatial instability of flow of a nonviscous, nonthermally conducting ordinary gas [1, 2]:

$$\frac{d\mathbf{u}}{dt}+\frac{1}{\rho}\nabla p=0,\ \frac{dp}{dt}+\rho c^2\operatorname{div}\mathbf{u}=0,\ \frac{dS}{dt}=0,\ \rho=\psi(p,S),\tag{1.1}$$

where **u** is the velocity vector; p, pressure; ρ, density; S, entropy; c, velocity of sound; t, time; $\mathbf{x}=(x, y, z)$, radius vector of a point in R^3; $\nabla=(\partial/\partial x, \partial/\partial y, \partial/\partial z)$; $d/dt=\partial/\partial t+\mathbf{u}\cdot\nabla$. The function ψ(p, S), which gives the equation of state of the ordinary gas, is assumed to be analytic.

A centered wave is a solution of the system (1.1) whose domain is covered by a single parameter family of acoustic characteristics passing through the given two-dimensional surface $\gamma_0\subset E^4=R^3\times R$ $(\mathbf{x}\in R^3,\ t\in R)$. In this case, the wave is said to be centered on γ_0.

In what follows, we examine the problem of a piston. Assume that the solution of system (1.1), satisfying the impermeability condition $\mathbf{u}\cdot\nabla h=0$ on Γ is given in a half space, whose boundary Γ is given by the equation $h(\mathbf{x})=0$ $(\nabla h\neq 0)$, is determined for $0\leqslant t\leqslant t_0$ This solution in what follows is called the unperturbed solution. A perturbation propagating along Γ arises at time t = 0 at the point Q ∈ Γ: the lateral wall begins to buckle according to a definite law so that outside the buckled part, it is given by the equation $h(\mathbf{x})=0$, while in the buckled part Γ', it is given by equation $h_1(\mathbf{x}, t)=0$. It is assumed that $h_1>0$ in the region occupied by the gas, $h_{1t}>0$ on Γ', and the surfaces $h(\mathbf{x})=0$ and $h_1(\mathbf{x}, 0)=0$ are tangent at the point Q. The intersection of Γ and Γ' forms an edge which moves according to a given law along Γ. Let γ_0 be a two-dimensional surface and E^4, traced out by this edge in time (Fig. 1 shows a picture illustrating the two-dimensional case). The unperturbed solution will describe a gas flow in the region bounded by the acoustic characteristic Γ_1 $(\varphi(\mathbf{x}, t)=0)$

$$\varphi_t+\mathbf{u}\cdot\nabla\varphi+c|\nabla\varphi|=0,\tag{1.2}$$

passing through γ_0 (φ > 0 in the region of unperturbed motion). It is necessary to find the

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